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SOME THEORETICAL PREDICTIONS OF MASS AND ELECTRON DENSITY OSCILLATIONS BASED ON A SIMPLE MODEL FOR TURBULENT WAKE MIXING

by

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1 - Introduction

In order to predict the radar backscatter of turbulent wakes of hypervelocity objects flying through the atmosphere, it is necessary to know some of the statistical properties of the dielectric constant of the turbulent medium. Correspondingly, the characteristics of the electron density fluctuations in the wake, and, in particular, the intensity of the electron density fluctuations must be known.[†]

Direct measurements of the electron density fluctuations in a turbulent wake plasma do not exist in the literature to date. Rothman et al.¹ have performed such measurements for the case of a rocket exhaust, using an ion probe and a microwave electron probe. In the case of wakes, however, only indirect and spatially unresolved measurements such as radar backscatter measurements² have been made. On the other hand, laboratory measurements^{3, 4} of the oscillations of mass density in the wake of hypersonic projectiles using shadow photographs and densitometry techniques, can determine the latter with fairly good spatial resolution. Thus, it would be very useful to be able to infer the electron density fluctuation characteristics in a wake from the corresponding fluctuations in mass density. In fact, the amplitudes of electron density and mass density fluctuations in wakes have sometimes been roughly equated in computations of wake radar cross-sections. As will be shown, such a procedure is not justified.

In the present paper, an attempt is made to relate the amplitude of mass and electron density fluctuations in a turbulent hypersonic wake on the basis of a simple model for the wake structure. The assumed wake structure is derived on the basis of physical arguments concerning the processes of turbulent wake mixing and growth. The model predicts large values of mass density fluctuations in the wake and is in that respect consistent with experimental observation. It similarly predicts wide differences in the intensities of mass and electron density oscillations, except in unusual circumstances.

The main virtue of the present model lies in its simplicity, rather than in its ability to make accurate predictions. It requires to be refined and extended (particularly to include chemical reactions) before it can be meaningfully applied to obtain quantitative results for hypersonic re-entry wakes.

2 - Structure and Growth of the Turbulent Wake

The present section is devoted to a discussion of a model for the turbulent wake core which represents the latter as consisting of two parts: a relatively hot portion of homogeneously mixed gas, and a relatively cold portion representing fluid engulfed by the expanding turbulent core, but which has not yet mixed on the molecular scale with the core gas. The physical arguments behind such a model are first discussed in Sec. 2.1, together with experimental evidence which may be adduced in favor of such a model. Section 2.2 then presents a simple analytical model of wake mixing which leads to the 'two-fluid' structure of the wake and introduces a 'lag-time' or 'lag-distance' for mixing of newly engulfed fluid with the remainder of the wake gas. An attempt is also made to relate this lag to kinetic energy of the wake turbulence.

2.1 - Physical Model of Wake Structure

The wake of hypersonic blunt bodies consists,⁵ at sufficiently high Reynolds numbers, of an inviscid outer wake and a viscous turbulent inner wake. This wake structure is well-known and need not be described here. It is sufficient to recall that the turbulent wake consists initially (i.e., at the neck) of the gas contained in the free shear layers shed from the body upon separation of the boundary layer and that it grows with downstream distance, and eventually engulfs the entire (originally inviscid) outer wake.

Experimental observations of turbulent wakes in hypervelocity ranges^{3, 4} and of turbulent subsonic wakes⁶, 7 reveal that the turbulent wake possesses a sharp front which separates it from the non-turbulent outside fluid. According to presently accepted notions in the theory of turbulence,⁸ the turbulent wake consists of eddies of widely varied sizes. Most of the kinetic energy of turbulence is contained in the relatively larger eddies, while the smaller eddies are responsible for most of the viscous dissipation of kinetic energy into heat. The very largest eddies do not contain much energy but they are important in contorting the wake surface. There is a continuous transfer of turbulence energy from eddies of a given size to smaller eddies, the rate of transfer depending on the eddy size. The large energy containing eddies have a relatively long lifetime, whereas the small energy dissipating

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⁺ The radar cross section depends in particular on the square of the mean value of electron density fluctuations.

eddies have much shorter lifetimes. In general, the eddy lifetime decreases rapidly with eddy size.

The growth of the turbulent wake and the manner in which it engulfs outer fluid may be broadly characterized as follows: The local propagation of the turbulent front bounding the wake core occurs by diffusion of vorticity fluctuations into the outer fluid.^{7,8} The diffusion is initiated by the smallest eddies, since they represent the greatest velocity gradients. The transfer of turbulence energy to the previously non-turbulent fluid is accomplished by the energy containing eddies, which transfer energy to the vorticity diffused into the outer fluid by the small eddies. In the above manner, turbulence is generated into previously quiescent fluid.

The mixing of outer fluid into the turbulent core, as distinguished from the transfer of turbulence energy, probably is due to large-scale mixing by the large eddies which distort the core boundary, though they do not themselves contain much energy. Only these eddies can be expected to be effective in mixing outer fluid into the wake, the smaller eddies serving to diffuse vorticity fluctuations.

The important features of the above admittedly superficial description of the turbulent wake and its growth are simply that the outer fluid is most probably mixed into the core by large scale eddies and that the turbulent front represents the boundary between different types of velocity fields and that consequently differences in composition or density between turbulent and quiescent fluid are not instantaneously erased as the wake engulfs the latter.

Thus, at least initially, the outer fluid is mixed into the wake in relatively large 'lumps', which contain vorticity fluctuations to a greater or lesser extent, but which retain, in particular, their initial relatively low temperature. This gives rise to a coarse-grained structure of the turbulent wake, which has been noted previously by Herlin and Hermann, 9 for instance.* The experiments of Slattery and Clay^{3, 4} appear to confirm such a wake model. Indeed, the large mass density fluctuations which they measure in the wake would be very difficult to explain on the basis of a well-mixed wake. Turbulent velocity, temperature, and pressure fluctuations can be reasonably expected to generate density fluctuations of no more than a few percent, whereas the observed values range up to ninety percent. A wake consisting of hot and cold lumps, on the other hand, can be expected to predict density fluctuations of the observed magnitude, as is shown in Section 3.

Similarly, the observations of Slattery and Clay that the correlation lengths of the mass den-

sity fluctuations in wakes are of the order of a body diameter can be considered to argue in favor of a coarse-grained wake structure.

The preceding considerations and interpretations of experimental results thus indicate that a wake structure consisting of a coarse grained structure of volumes of newly engulfed outer fluid embedded in a more homogeneous core is at least plausible. In the next sub-section, a very simple model of such a coarse-grained wake is discussed.

2.2 - Simple Wake Mixing Model

In the preceding sub-section, it was argued that relatively large lumps of inviscid fluid are mixed into the turbulent wake core as the latter grows behind a body. The subsequent mixing of this entrained gas in a more intimate manner with the core gas depends on the turbulence intensity and molecular diffusion in the wake, or more precisely, on the eddy and molecular diffusion coefficients. When the intensity of turbulence is high, the redistribution of inhomogeneities in the wake is effected essentially by random convection by the turbulent field, for inhomogeneities whose scale is not below the "cut-off" scale for velocity fluctuations set by viscosity effects. The effects of molecular conduction and diffusion are negligible for inhomogeneities of such a scale. Below the "cutoff" scale of turbulence, however, molecular effects dominate and act to erase small scale inhomogeneities. In the absence of molecular diffusion and conduction, the effect of eddy diffusivity (or random convection) is to break up large volume elements into smaller ones (down to the cut-off scale) and, therefore, to increase mean gradients rather than decrease them. The increase in mean gradients leads however to increased molecular dissipation, so that eddy diffusion indirectly leads to dissipation of inhomogeneities.

The process of mixing in the early portions of the wake where the turbulence intensity is high may therefore be approximately viewed as follows: The newly entrained, relatively large volume elements in the wake are broken up into ever-smaller elements by random convection. This break up characteristic of turbulence, proceeds without appreciable molecular effects until the size of the volume elements and the resulting gradients are such that molecular effects become important and erase the inhomogeneities, leading to a well-mixed structure. If the 'take-over' of molecular effects is sufficiently sharp, there will be a negligible fraction of gas which is neither unmixed nor completely mixed (on the molecular level, that is), and the core will consist of gas in essentially two states only, namely the unmixed state corresponding to the state of the newly entered fluid, and the mixed state, corresponding to the state of the gas which has been within the turbulent core long enough to be thoroughly assimilated.

In the far wake, where the turbulent velocities have presumably died down, the above model

^{*}Herlin and Hermann refer to the structure as a "marble-cake" structure.

is not expected to hold. The eddy diffusivity is probably no longer very important, and the break up of the lumps no longer occurs very efficiently. The temperature inhomogeneities then decay essentially by conduction. In fact, sufficiently far downstream all velocities will have essentially died down. Thus, the final steps of thermal equalization proceed purely by molecular diffusion. The limits of application of the 'two-fluid' model is undoubtedly dependent on Reynolds number and body geometry, and may not be applicable at all when the turbulent intensity is very low, corresponding to very small Reynolds numbers.

The net effect of the mixing process described above is to eventually mix fluid entering the turbulent core homogeneously with the core gas. The mixing does not occur, however, instantaneously as the fluid enters inside the turbulent front, but after some lag in time and therefore downstream distance, which depends on the intensity of turbulence and on the molecular viscosity and diffusivity in the wake. The above mixing model is therefore intermediate between the homogeneous mixing model which assumes instantaneous mixing, and the inviscid random convection model of Obukhof¹⁰ and Corrsin, 11 in which it is assumed that molecular mixing does not occur at all. The two above extremes correspond to a zero and infinite lag respectively. A quasi-one dimensional model for (chemically reacting) turbulent wakes has been analyzed by Lin and Hayes¹² for the extreme cases of homogeneous mixing and inviscid random convection. A set of equations describing the chemical reactions and mixing in the wake for finite lags can similarly be written and constitutes a simple generalization of the work of Lin and Hayes.

2.3 - Equations Describing Quasi-One Dimensional Model for Wake Mixing with Lag

The model of a wake consisting of homogeneously mixed 'old' fluid and unmixed 'new' fluid in various stages of break up can be adequately described by a quasi-one dimensional wake model, in which the mean turbulent wake boundary is assumed to be specified. The model is illustrated in Fig. 1. The fluctuations in the turbulent front are ignored. The growth of the turbulent wake is represented by the flux of outer inviscid fluid into the turbulent core through the wake boundary. The entering fluid then forms part of the unmixed portion of the core for a mean distance & (measured in body diameters), after which it is homogeneously mixed with the remainder of the wake by molecular effects. The actual mixing of fluid entering at a particular station in the wake need not occur suddenly after a distance (, but may be spread over a (small) range of lags centered about ζ .

The turbulent wake is then characterized by the pressure (assumed constant across the wake), and by the velocity, density, enthalpy and chemical composition (if chemistry is included) of the 'hot' homogeneously mixed and 'cold' unmixed portions of the wake, together with the relative fractions of each, as a function of downstream distance. The values of the velocity, density, enthalpy, and chemical composition of the inviscid gas at the edge of the turbulent core are assumed specified, together with the total wake width or area. The wake evolution is then described by the onedimensional equations of conservation of mass, momentum and energy, by corresponding equations describing the mixing lag, and by the equations expressing the chemical reactions in the wake, or the thermodynamic characteristics of the gas (equations of state) if chemical reactions can be ignored. In the latter case, which is applicable to wakes of relatively low-speed projectiles in ballistic ranges for instance, the equations specifying the quasi-one dimensional model are the following (z denotes downstream distance):

Conservation of Mass:

$$\frac{\mathrm{d}}{\mathrm{d}z} \left(\rho_{\mathrm{c}} \mathrm{u}_{\mathrm{c}}^{\mathrm{A}} \mathrm{c} \right) + \frac{\mathrm{d}}{\mathrm{d}z} \left(\rho_{\mathrm{h}} \mathrm{u}_{\mathrm{h}}^{\mathrm{A}} \mathrm{h} \right) = \rho_{\mathrm{i}} \mathrm{u}_{\mathrm{i}} \frac{\mathrm{d}A}{\mathrm{d}z} \qquad (1)$$

Conservation of Momentum:

$$\frac{d}{dz} \left(\rho_{c} u_{c}^{2} A_{c} \right) + \frac{d}{dz} \left(\rho_{h} u_{h}^{2} A_{h} \right) - A \frac{dp}{dz} = \rho_{i} u_{i}^{2} \frac{dA}{dz}$$

$$\dots (2)$$

Conservation of Energy:

$$\frac{d}{dz} \left[\rho_{c} u_{c} \left(h_{c} + \frac{u_{c}^{2}}{2} \right) A_{c} \right] + \frac{d}{dz} \left[\rho_{h} u_{h} \left(h_{h} + \frac{u_{h}^{2}}{2} \right) A_{h} \right] = \rho_{i} u_{i} H \frac{dA}{dz}$$
(3)

Lag Equation in Terms of Mass:

$$\frac{d}{dz} \left(\rho_{h} u_{h} A_{h} \right) = \rho_{i} u_{i}^{\prime} \frac{dA^{\prime}}{dz}$$
(4)

Lag Equation in Terms of Momentum:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\rho_{\mathrm{h}}u_{\mathrm{h}}A_{\mathrm{h}}\right) - A_{\mathrm{h}}\frac{\mathrm{d}p}{\mathrm{d}z} = \rho_{\mathrm{i}}^{1/2}\frac{\mathrm{d}A^{\mathrm{i}}}{\mathrm{i}\frac{\mathrm{d}A^{\mathrm{i}}}{\mathrm{d}z}}$$
(5)

Lag Equation in Terms of Energy:

$$\frac{d}{dz} \left[\rho_h u_h \left(h_h + \frac{u_h^2}{2} \right) A_h \right] - T = \rho_i u_i' H \frac{dA'}{dz}$$
... (6)

Equations of State (assuming a perfect gas):

$$\rho_{\rm c} \mathbf{h}_{\rm c} = \rho_{\rm h} \mathbf{h}_{\rm h} = \frac{\gamma}{\gamma - 1} \mathbf{p} \tag{7}$$

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In the above equations, ρ_{i} is the density of the inviscid gas at the edge of the turbulent core, u_i is its velocity, H its total enthalpy $(H = h_i + \frac{u_i}{2})$, and p is the pressure. They are all assumed known, except for the pressure.

The density, velocity, and static enthalpy h for the hot and cold portions of the turbulent wake are denoted by the subscripts h and c, respectively. The areas A and A are the 'partial areas' of hot and cold fluid, and their sum is equal to the wake area A:

$$A_{h} + A_{c} = A .$$
 (8)

All unprimed quantities are evaluated at the station z, and all primed quantities are evaluated at the station $z - \zeta$.

Equations (4), (5), and (6) simply express the fact that outer fluid entering the turbulent wake at station $z - \zeta$ is homogeneously mixed with, and becomes part of the hot gas at station z.

The term T in Eq. (6) represents the transfer of heat between the hot and cold gases in the wake by molecular conduction. If T is set equal to zero, the energy conservation equation and its lag counterpart are greatly simplified and reduce to the equations:

$$h_{c} + \frac{u_{c}^{2}}{2} = h_{h} + \frac{u_{h}^{2}}{2} = H$$
. (3a)

If chemical reactions are included a set of equations describing the reactions must be used, which exhibits the feature that chemical reactions proceed at different rates before and after mixing.

The lag distance ζ must be determined in order to obtain a solution to the set of Eqs. (1) through (7), or the corresponding equations including chemistry.

The mixing lag distance ζ is equal to some lag time τ multiplied by the local wake velocity u_w , i.e., $\zeta = u_w \tau$. The lag time τ represents the time required for the turbulent velocity field to break down the initial lumps of inviscid fluid entering the wake to the size at which molecular diffusion takes over. Thus, it is not unreasonable to assume that τ is proportional to the characteristic time for the transfer of turbulence energy from the scale of the initial lumps to the turbulence cut-off scale. Combining this assumption with what is known of that characteristic time for subsonic wakes and with the one-third power law for wake growth it is possible to conclude* that the lag distance is an increasing function of wake downstream distance, and is in fact approximately proportional to downstream distance along the wake (in a coordinate system fixed in the body), when the intensity of turbulence is sufficiently high so that the eddy diffusivity is very much larger than the molecular diffusivity. For lower turbulence intensities, the lag distance can be expected to increase more sharply with downstream distance.

- 3	-V	lass	and	Densit	y Os	scilla	ations
In	the	Mix	ing-	with-La	ig W	lake	Model

The mean values of the mass and electron density variations predicted by the present wake model can now be discussed. Chemical reactions will not be considered in the present discussion. The wake structure is therefore specified, at any axial distance, by specifying the fractions of mixed and unmixed fluids and by specifying their densities or temperatures. Here it is assumed, once more, that each of the two portions is characterized by a single temperature or density.

3.1 – Mean Quantities and Mean Deviations for the Mixing-with-Lag Wake Model

Consider a turbulent wake consisting of a granular mixture of the same fluid at a given pressure but at two different temperatures T_{h} and T_{c} .

The pressure is assumed constant everywhere, and T $_{\rm c}$ represents the temperature of the relatively

cold gas recently engulfed in the turbulent core, while T_{h} is the temperature of the hotter homoge-

neous portion of the core gas. The corresponding gas densities are $\rho_{\rm h}$ and $\rho_{\rm c}$, and if the wake is

ionized the corresponding electron densities are $n_{e_{h}}$ and $n_{e_{c}}$. For the purpose of the present dis-

cussion, ionization equilibrium is assumed. In general, all quantities Q characterizing the wake fluid will have two values, $Q_{\rm h}$ and $Q_{\rm c}$ in the wake.

Let V_h and V_c be the partial volumes of hot

and cold gas in a given slice of the wake of volume V. The mean value and root mean deviations of any quantity Q characterizing the wake are determined within that slice by the extreme values of Q, i.e., Q_{h} and Q_{c} , and by the ratio $\epsilon = V_{c}/V$ which represents the fraction of cold gas in the wake.⁺ It is important to note that the distribution of the hot and cold portions is not important in computing the mean values and root mean deviations of a quantity Q in the wake, once ϵ is specified. This point is illustrated in Fig. 2. Fig. 2a depicts a representative variation of, say the mass density ρ along any

a forthcoming report in which solutions describing the present wake model are also presented.

 $+\epsilon$ may also be interpreted as the mean fraction of the time during which a given point in the wake slice slice is immersed in 'cold' gas.

A more detailed discussion of the arguments leading to the result given here is presented in line in the wake slice. The various lengths of the cold (high density) portions represent the various stages of break up of unmixed fluid which has recently entered the turbulent boundary. Fig. 2b shows a distribution of hot and cold fluid in which all the separate hot (and cold) portions have been lumped together, while ϵ remains unchanged. This distribution predicts the same mean value and root mean deviations for any quantity Q and is therefore equivalent to the one in Fig. 2a for the purposes of the present computations.

The relations derived below for the mean value and root mean deviation of wake properties become rather obvious by reference to Fig. 2b.

et
$$\tau_{Q}$$
 be the ratio

$$\tau_{Q} = \begin{cases} Q_{c}/Q_{h} & \text{if } Q_{c} \geqslant Q_{h} \\ Q_{h}/Q_{c} & \text{if } Q_{c} \leqslant Q_{h} \end{cases}$$
(9)

of the larger to the smaller value of Q in the wake, so that $\tau_{\rm Q} \ge 1$.

The mean value of Q is defined as

L

$$\overline{\Omega} = \frac{1}{V} \int_{V} \Omega \, dV = \epsilon \Omega_{c} + (1 - \epsilon) \Omega_{c}$$
(10)

Similarly, the mean square deviation of Q from the mean is:

$$\Delta Q^{2} = \frac{1}{V} \int_{V} (Q - \overline{Q})^{2} dV$$
$$= \epsilon (Q_{c} - \overline{Q})^{2} + (1 - \epsilon) (Q_{h} - \overline{Q})^{2}. (11)$$

From Eqs. (9), (10) and (11) we may write

$$\left(\frac{\Delta Q}{\overline{Q}}\right)^{2} = \frac{\epsilon (1-\epsilon) (\tau_{Q-1})^{2}}{\left[1+\epsilon (\tau_{Q-1})\right]^{2}} \quad \text{if } Q_{c} > Q_{h}.$$

$$\left(\frac{\Delta Q}{\overline{Q}}\right)^{2} = \frac{\epsilon (1-\epsilon) (\tau_{Q-1})^{2}}{\left[1+(1-\epsilon) (\tau_{Q-1})\right]^{2}} \quad \text{if } Q_{c} < Q_{h}.$$

$$\dots (12a)$$

$$\dots (12b)$$

Equation (12) a or b expresses a necessary relationship between the mean square deviation of Q, the ratio of its extreme values Q_{c} and Q_{h} and the

relative proportion ϵ of cold fluid in the wake. The only requirement for its validity is that all but a negligible portion of the wake be represented by the values Ω_c and Ω_h of Ω .

* The evolution of an initially large lump of cold fluid is depicted schematically in Fig. 2d.

<u>3.2 – Application to Mass and Electron Density</u> Variations and Their Inter-relations

Letting $Q \equiv \rho$, the mass density in the wake, Eq. (12a) applies, since $\rho_c > \rho_h$ and

$$\left(\frac{\Delta\rho}{\overline{\rho}}\right)^{2} = \frac{\epsilon (1-\epsilon) (\tau_{\rho}-1)^{2}}{\left[1+\epsilon (\tau_{\rho}-1)\right]^{2}}$$
(13)

In addition, assuming the perfect gas law to be approximately valid (chemical reactions, as has been mentioned above, are neglected), the value of τ_0 is:

$$\tau_{\rho} = \frac{\rho_{\rm c}}{\rho_{\rm h}} \approx \frac{T_{\rm h}}{T_{\rm c}} \tag{14}$$

Letting $Q \equiv n_e$, the electron density in the wake, Eq. (12b) yields

$$\left(\frac{\Delta n_{e}}{\overline{n}_{e}}\right)^{2} = \frac{\epsilon (1-\epsilon) (\tau_{e}-1)^{2}}{\left[1+(1-\epsilon) (\tau_{e}-1)\right]^{2}}$$
(15)

since n < n =. In fact, because the degree of c = h

ionization varies quite rapidly with temperature, it may be assumed that $n \underset{c}{e} \ll n \underset{h}{e}$, and consequently

$$\boldsymbol{\tau}_{\mathbf{e}} = \frac{{}^{\mathbf{n}} \mathbf{e}_{\mathbf{h}}}{{}^{\mathbf{n}} \mathbf{e}_{\mathbf{c}}} \gg 1 .$$
 (16)

Thus, $\left(\frac{\Delta n_e}{\overline{n}_e}\right)^2$ may be approximated by its limiting value for infinite τ_c :

$$\left(\frac{\Delta n_e}{\overline{n}_e}\right)^2 \approx \frac{\lim_{\tau_e \to \infty} \left(\frac{\Delta n_e}{\overline{n}_e}\right)^2}{\tau_e \to \infty} \left(\frac{\Delta n_e}{\overline{n}_e}\right)^2 = \frac{\epsilon}{1-\epsilon} \quad . \tag{17}$$

Note that for a given value of ϵ , $\left(\frac{\Delta n_e}{\frac{n}{e}}\right)^2$ is a mono-

tonically increasing function of r_e , so that the limiting value in Eq. (17) is an upper limit to

$$\left(\frac{\Delta n_e}{\overline{n}_e}\right)^2 \text{ for a given value of } \epsilon.$$

The equations for
$$\left(\frac{\Delta\rho}{\overline{\rho}}\right)^2$$
 and $\left(\frac{\Delta n_e}{\overline{n}_e}\right)^2$ derived

above indicate that these two quantities have, in $P(x) = \sum_{i=1}^{n} P(x) = \sum_{i=1}^{n}$

general, quite different values in a wake. In particular, it may be noted that low values of ϵ tend to make the mean mass density oscillations large compared to the mean electron density oscillations and that high values of ϵ have a contrary effect. This is simply due to the fact that the mass and electron densities are 'inversely' related, so to speak, since a high mass density implies a low electron density and vice versa.* In general, it can be expected that a relatively small fraction of the turbulent wake will consist of unmixed fluid. It follows that the mean square electron density variations in the wake will be smaller than the corresponding mass density oscillations.

The dependence of
$$\left(\frac{\Delta \rho}{\overline{\rho}}\right)^2$$
 and $\left(\frac{\Delta n_e}{\overline{n_e}}\right)^2$ on

 ϵ and τ is depicted in Fig. 3, from which it can be seen, for instance, that for values of τ_{ρ} of about 3

or more
$$\left(\frac{\Delta n}{n_e}\right)^2$$
 is less than $\left(\frac{\Delta \rho}{\overline{\rho}}\right)^2$ for $\epsilon \leq 0.2$ and
that when $\tau_{\rho} \ge 5$, $\left(\frac{\Delta n}{n_e}\right)^2$ is less than $\left(\frac{\Delta \rho}{\overline{\rho}}\right)^2$ for
 $\epsilon \ne 0.4$. Note that such values of $\tau_{\rho} = \frac{T_h}{T_o}$ can be

expected in the wakes of high speed re-entry vehicles.

of the mass density oscillations.

The ratio of mean square electron to mass density oscillations in the wake as a function of the temperature ratio $\frac{T_h}{T_c} \approx \tau_{\rho}$ is shown in Fig. 4 for various values of ϵ . From that figure it can be seen that the value of the mean square electron density oscillations for $\tau_{\rho} = 5$ and $\epsilon = 0.1$ for instance is almost an order of magnitude below that

So far, the discussion of mass and electron density fluctuations has dealt with their dependence on the fraction ϵ of unmixed fluid in the turbulent wake core, and on the ratio of temperatures T_{c} and T_{h} of the relatively cold unmixed gas and of the hotter homogeneously mixed gas. The actual values of those quantities and their variation with downstream distance have not been specified. In fact, a determination of those quantities for the present wake mixing model requires the solution of the set of equations (1) through (7) which describe the mixing. However, a simple estimate of the mean value of the mass density oscillations and its variation with downstream distance may be obtained by using existing solutions 5 for the axis and edge enthalpies in hypersonic wakes based on homogeneous mixing models and some estimate for the mixing lag distance ζ.

The axis and edge temperatures can be used to represent $T_{\rm h}$ and $T_{\rm c}$. The wake cooling in homo-

geneous mixing models is primarily due to the admixture of cold outer fluid as the wake grows. Thus, to a first approximation, a lag in homogeneous mixing of newly engulfed gas can be expected to result in a corresponding lag in the core temperature decay. Thus, the core temperature T_h for the model of mixing with lag can be approximately obtained from the instantaneous mixing solution by displacing the latter by the lag distance. The unmixed portion of the cold wake at z consists

of fluid which entered the turbulent wake between $z - \zeta$ and z. Therefore, the temperature T_c of the

'cold' portion of the turbulent wake at a given downstream location z can be approximately taken to be the mean value of the temperature of the inviscid fluid at the turbulent wake edge, averaged over a distance behind the given station equal to the lag distance ζ .

The axis and edge temperatures for the turbulent wake of a sphere traveling at Mach number 8.5, as computed by Hromas¹³ are depicted in Fig. 5, together with the resultant 'hot' and 'cold' temperatures obtained by introducing a mixing lag, as discussed above. The lag distance selected in Fig. 5 is $\zeta = \beta z$, with $\beta = 0.3$. In other words, the lag distance z is assumed to increase linearly with z, in accordance with the discussion of the variation of ζ in Section 2. The choice of the value of the coefficient β is essentially arbitrary at this

point, and in computing values o

$$f\left(\frac{\Delta p}{\overline{\rho}}\right)$$
 for the

mixing-with-lag model several values of β have been chosen (cf Fig. 6).

The value of ϵ at any station is determined by the ratio of the volume of gas which entered the turbulent boundary between z and z - ζ to the volume of gas present in the wake at z. If the wake growth is known, ϵ may be computed approximately as

$$\epsilon = \frac{\int_{z-\zeta}^{z} dA(z)}{A(z)} = \frac{A(z) - A(z-\zeta)}{A(z)}, \qquad (18)$$

where A is the mean wake cross section, $A = \frac{\pi b^2}{4}$ (b is the wake diameter). Using a one-third power law fitted to experiment to obtain A (z) and using the hot and cold temperatures obtained as dis-

cusses previously, the values of $\left(\frac{\Delta\rho}{\overline{\rho}}\right)$ predicted

by the wake mixing model for various values of the mixing lag ζ are shown in Fig. 6 for the case of the 8.5 Mach number sphere wake. Also shown in that figure are experimental determinations of

 $\left(\frac{1\rho}{\delta}\right)$ obtained by Slattery and Clay⁴ for very nearly

^{*}The electron density distribution corresponding to the mass density distribution shown in Fig. 2b is shown in Fig. 2c.

the same velocity. It may be noted that the theoretical predictions give good agreement in terms

of the shape of the $\left(\frac{\Delta\rho}{\overline{\rho}}\right)$ curve, although they do

not predict the experimental amplitudes and are flatter than the experimental curve. Note also that the maxima in all the theoretical curves occur virtually at the same downstream location, which is very close to the apparent maximum of the experimental curve. The agreement is regarded as quite encouraging, considering the crudeness of the computations.

4 - Concluding Remarks

In the preceding sections, a simple model fo the turbulent wake in which the latter exhibits a granular structure consisting of a mixture of 'hot' and 'cold' elements of fluid of various sizes was investigated. Such a structure was justified on the basis of physical arguments concerning the growth of the turbulent wake by engulfment of the neighboring inviscid wake gas. The wake mixing was described in terms of a mixing lag distance ζ which represents the mean distance traveled by fluid entering the turbulent boundary before it is homogeneously mixed with the turbulent core gas. The model appears to be at least partly supported by the experimental evidence of large mass density oscillations in the wake, and of large values of the correlation lengths of those oscillations. Indeed, the model predicts a variation with downstream distance of the amplitude of the mass density oscillations in the wake quite similar to that observed experimentally. The model also leads to the determination, through elementary computations, of relations between electron and mass density oscillations in wakes when local thermodynamic equilibrium obtains in the wake. In particular, it was shown that the mean square electron and mass density variations in the wake are, in general, different and that the former are generally significantly lower than the latter.

The application of the model to the prediction of mean values and mean oscillations of the mass and electron densities in wakes of hypersonic high speed re-entry bodies requires the inclusion of chemical reactions in the wake model. A formal quasi-one dimensional formulation of the equations describing the model was discussed. In view of the initial success of the simple calculations performed in the present paper, it is felt that a more complete investigation of the wake-mixing-with-lag model including chemical reactions is worthwhile.

The encouraging results of the simple calculations also suggest simultaneous experimental

determinations of $\left(\frac{\Delta \rho}{\overline{\rho}}\right)$ and $\left(\frac{\Delta n}{\overline{n}}_{e}\right)$ to provide a

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- Pippert, G. F., "Determination of the Correlation Length for Electromagnetic Scattering from Wallops Island Results," <u>Semi-Annual</u> <u>Technical Summary Report to the Advanced</u> <u>Research Projects Agency</u>, II-25 to II-29, Lincoln Laboratory, Lexington, Mass., March 1963.
- Slattery, R. E., and Clay, W. G., "Re-entry Physics and Project PRESS Programs," <u>Ibid.</u>, III-9 to III-25, February 1962.
- Slattery, R. E., and Clay, W. G., "The Turbulent Wake of Hypersonic Bodies," paper no. 2673-63, <u>ARS 17th Annual Meeting</u>, Los Angeles, California (13-18 November 1962).
- Lees, L., and Hromas, L., "Turbulent Diffusion in the Wake of a Blunt Nosed Body at Hypersonic Speeds," <u>J. Aero/Space Sci.</u>, <u>29</u>, 976-93, August 1962.
- Townsend, A. A., <u>The Structure of Turbulent</u> <u>Shear Flow</u>, Cambridge Univ. Press, Cambridge Univ. Press, Cambridge (1956).
- Corrsin, S., and Kistler, A. A., "Free-Stream Boundaries of Turbulent Flows," NACA Report 1244, Washington, 1955.
- Hinze, J. O., <u>Turbulence</u>, McGraw-Hill Book Company, Inc., New York (1959).
- Herlin, M. A., and Herrmann, J., <u>Semi-Annual Technical Summary Report to the</u> <u>Advanced Research Projects Agency</u>, II-4, Lincoln Laboratory, Lexington, Mass., March 1963.
- Obukhof, A. M., Izv. Akad. Nauk, USSR, Geogr. i Geofiz., <u>13</u>, 58, (1949).
- 11. Corrsin, S., J. Appl. Phys., 22, 469 (1951).
- Lin, S. C., and Hayes, J. E., "A Quasi One-Dimensional Model for Chemically Reacting Turbulent Wakes of Hypersonic Objects," <u>Research Report 157</u>, AVCO-Everett Research Laboratory, July 1963.
- Hromas, L., Space Technology Laboratories, Inc., Private Communication, October 1963.

direct test of the validity of the model.

References

 Rothman, H. S., Guthart, H., and Morita, T. "Measurement of the Spectrum of Ion and Electron Fluctuations in a Thermally Produced Turbulent Plasma" (U). Paper presented at the AMRAC Meeting in Annapolis, (22-24 October 1963).

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Wake Mixing with Lag. The fluid entering the mean turbulent boundary at 2 - j travels a distance j before mixing on the molecular level with the gas already present in the wake core. Thus the change in the core density and temporature over the distance dz is controlled by the fluid entering the turbulent core within $d\{z - j\}$. Fig. 1



Variation of mean square mass and electron density oscillations with the cold component fraction Ξ in the coarse grained wake model. The ratio of cold to hot gas donsities is used as a parameter. Fig. 3



Equilibrium temperatures along the axis and front of the turbulent wake of a sphere computed according to a homogeneous mixing model and corresponding 'hot' and 'cold' temperatures for the mixing-with-lag wake model. Fig. 5

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Ratio of mean square electron density to mass density oscillations as a function of the density ratio of the cold and hot wake components in the coarse-grained wake model. The fraction E of cold fluid is used as a parameter. Fig. 4



Root mean square density oscillations predicted by mixing-with-lag wake model for a sphere at flight Mach number 8.5 and comparison with experiment. Fig. 6